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AERODYNAMIC FORCES ON A CYLINDER FOR THE FREE
MOLECULE FLOW OF A NON-UNIFORM GAS

by
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FLUID FLOW AND
HEAT TRANSFER
AT LOW PRESSURES
AND TEMPERATURES

AERODYNAMIC FORCES ON A CYLINDER FOR THE FREE
MOLECULE FLOW OF A NON-UNIFORM GAS

By

S. HELL

and

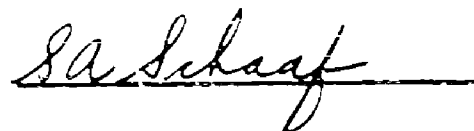
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A handwritten signature in cursive script, appearing to read 'S. A. Schaap', is written over a horizontal line.

N O M E N C L A T U R E

dA	= element of surface area
C_D	= drag coefficient, Eq. 3.4
C_L	= lift coefficient, Eq. 3.5
$C_{D_{pxx}}, C_{L_{xy}}, \text{etc.}$	= partial drag and lift coefficients proportioned to term indicated in subscript, Eqs. 3.4, 3.5
dE_i, dE_r, dE_w	= energy fluxes incident, reflected and diffusely reflected from dA , Eq. 2.7
F_D	= drag force per unit length of cylinder
F_L	= lift force per unit length of cylinder
$d\vec{F}$	= force on dA
$d\vec{F}_i$	= component of $d\vec{F}$ due to incident molecules
$d\vec{F}_r$	= component of $d\vec{F}$ due to reflected molecules
f	= molecular velocity distribution function, Eq. 2.5
f^0	= Maxwellian distribution, Eq. 2.4
I_0, I_1	= modified Bessel functions of zero and first order
k	= thermal conductivity
M	= Mach number
m	= molecular mass
N_w	= number of molecules incident on dA per unit time
p	= pressure
p_i, p_r, p_w	= pressure on dA due to incident, reflected, and diffusely reflected molecules, respectively, Eq. 2.10
p_{xx}, p_{yy}, p_{zz}	= normal stress deviation terms, Eqs. 2.5, 2.6
q_x, q_y, q_z	= heat flux terms, Eqs. 2.5, 2.6
r	= cylinder radius
R	= gas constant
S	= molecular speed ratio, Eq. 3.3
S_w	= molecular speed ratio referred to cylinder temperatures, Eq. 3.3 (S_w assumed equal to S in computations)
T	= gas temperature
T_i	= temperature of incident molecules, Eq. 2.13
T_w	= cylinder temperature
u, v, w	= molecular velocity component in x, y, z direction, Fig. 1
U, V, W	= gas velocity components
u', v', w'	= molecular velocity components in x', y', z' directions, Fig. 1

NOMENCLATURE (CONT'D)

- x, y, z = coordinates referred to flow direction, Fig. 1
 x', y', z' = coordinates referred to surface element, Fig. 1
 α = thermal accommodation coefficient, Eq. 2.7
 γ = ratio of specific heats
 μ = viscosity coefficient
 θ = angle between surface normal and flow direction, Fig. 1
 ρ = gas density
 $\bar{\sigma}, \bar{\sigma}'$ = reflection coefficients for tangential and normal momentum transfer, Eqs. 2.9, 2.10
 τ_i, τ_r, τ_w = shear stress due to incident, reflected and diffusely reflected molecules, respectively, Eq. 2.9
 $\tau_{xy}, \tau_{xz}, \tau_{yz}$ = shear stress terms, Eqs. 2.5, 2.6

AERODYNAMIC FORCES ON A CYLINDER FOR THE FREE
MOLECULE FLOW OF A NON-UNIFORM GAS

1.0 INTRODUCTION

The forces exerted upon a body in free molecule flow, i.e., on a body whose characteristic dimension is small compared to the molecular mean free path, has been determined theoretically by Tsien (Ref. 1), Stalder and Zurick (Ref. 2) and others (Refs. 10 to 12). Experiments by Stalder, Goodwin and Creager (Ref. 3), and by Kans and Estermann (Ref. 4) have provided general confirmation of the theoretical developments.

These investigations have been confined to the case in which the gas flow past the body is in Maxwellian equilibrium. This restriction is satisfactory for free flight considerations but not for what may, perhaps, prove to be one of the most important applications of free molecule flow analyses. This is wind tunnel measurement of the velocity, and perhaps other flow variables, of a gas stream by means of small "free molecule flow" probes. The possibilities are especially promising in connection with hypersonic or supersonic wind tunnels operating at low test section densities. These probes might be of either the heat transfer or aerodynamic force type, or perhaps both. It is clear, however, that for probing boundary layers or shock waves or other characteristic parts of a flow field, the instrument will often be exposed to a gas stream which is not in Maxwellian equilibrium. The occurrence of a large shear stress in a boundary layer, for example, produces a departure of the molecular velocity distribution from the Maxwellian. As will be seen below, this non-uniformity gives rise to additional force terms.

Approximate analyses of the forces on bodies in such non-Maxwellian free molecule flows have been made by Einstein (Ref. 5) and Epstein (Ref. 6) in connection with the radiometer problem. In their discussions the non-uniformity was confined to the case of a local heat flux, and in any case the results were only of order-of-magnitude validity.

In the present report a more detailed analysis of the free molecular flow of a non-uniform gas is made. The results are general

enough to include arbitrary (small) stress and heat flux. The body geometry has been confined to that of a right circular cylinder oriented perpendicular to the gas flow velocity, and it is assumed the body's thermal conductivity is so large that it is at uniform temperature. Only the aerodynamic force characteristics are determined. The extensions to the determination of heat transfer characteristics, to other body geometries and to the case of very low internal conductivity, will be presented in subsequent reports. The method, however, will be apparent from the present paper.

2.0 METHOD OF ANALYSIS

2.1 The Incident Molecules

The general method followed is a direct extension of the method for the Maxwellian case used by Tsien (Ref. 1) and Stalder and Zurick (Ref. 2), and was suggested by Goodwin. The force per unit area $d\vec{F}/dA$ on a surface element dA is broken up into a part $d\vec{F}_i/dA$ due to the incident molecules, and a part $d\vec{F}_r/dA$ due to the reflected or re-emitted molecules. Total force characteristics are obtained by integration over the surface of the body.

The force $d\vec{F}_i/dA$ is analysed in terms of the component p_i normal to the surface (pressure) and the component τ_i tangential to the surface (shear stress). See Fig. 1. These quantities are obtained in terms of the molecular velocity distribution function, $f(u, v, w)$, which is the number density per unit volume of phase space of molecules with velocity components u, v, w . One has,

$$p_i = m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} u'^2 f \, du' \, dv' \, dw' \quad (2.1)$$

$$\tau_i = m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} u' v' f \, du' \, dv' \, dw' \quad (2.2)$$

where m is the molecular mass. The velocity components u, v, w and u', v', w' are related by (see Fig. 1),

$$\left. \begin{aligned} u &= -u' \cos \theta + v' \sin \theta \\ v &= -u' \sin \theta - v' \cos \theta \\ w &= w' \end{aligned} \right\} \quad (2.3)$$

where θ is the angle between the (outward) normal to the surface and the x (flow) direction.

The function f is given in the Maxwellian case by

$$f(u,v,w) = f^0 = \frac{\rho}{m(2\pi RT)^{3/2}} e^{-\frac{(u-U)^2 + v^2 + w^2}{2RT}} \quad (2.4)$$

where ρ, T, U, R are the gas density, temperature, flow velocity and the gas constant, respectively. For the non-Maxwellian case, one uses the distribution function considered by Maxwell (Ref. 7) and Grad (Ref. 8):

$$f = f^0 \left[1 + \frac{1}{2\rho RT} \left\{ p_{xx}(u-U)^2 + p_{yy}v^2 + p_{zz}w^2 - 2\tau_{xy}(u-U)v - 2\tau_{xz}(u-U)w - 2\tau_{yz}vw - 2\left(1 - \frac{(u-U)^2 + v^2 + w^2}{5RT}\right)(q_x(u-U) + q_yv + q_zw) \right\} \right] \quad (2.5)$$

where p_{xx}, τ_{xy}, q_x , etc. are the viscous stress and heat flux components. These quantities, together with ρ, U and T , are treated as constants throughout the region occupied by the cylinder. For the equilibrium case, and for small gradients of velocity and temperature, these terms are related to the velocity and temperature by the Navier-Stokes relations

$$\left. \begin{aligned} p_{xx} &= -2\mu \left[\frac{\partial U}{\partial x} - \frac{1}{3} \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right) \right] \\ \tau_{xy} &= \mu \left[\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right] \\ q_x &= -K \frac{\partial T}{\partial x} \end{aligned} \right\} \quad (2.6)$$

For rapid processes (e.g., shock waves) or for large gradients of temperature and velocity more complicated relations exist; see Ref. 8 for details. In the present report, the results will be obtained in terms of the variables listed in Eq. 2.5, so that they are independent of whether or not Eqs. 2.6 are valid. It should also be

mentioned that for sufficiently non-uniform situations the approximation of Eq. 2.5 itself becomes poor. Additional terms in the expression for the molecular velocity distribution would then be required and would contribute to the gross force exerted upon the body. The relative dependence upon the parameters $p_{xx}, \tilde{u}_{xy}, q_x$ etc., however, would be unchanged.

2.2 The Reflected Molecules

The determination of $d\vec{F}_r$, the force on dA due to the re-emitted molecules, requires a specification of the nature of the interaction of the impinging molecules with the surface. The treatment of this question will be somewhat generalized in this report from the usual treatment of Maxwell (Ref. 7) which has been followed in the subsequent work of Tsien (Ref. 1), Stalder and Zurick (Ref. 2), and others. Traditionally, the interaction has been described in terms of two parameters, the thermal accommodation coefficient α and the specular reflection coefficient σ . The quantity α is defined by:

$$\alpha = \frac{dE_i - dE_r}{dE_i - dE_w} \quad (2.7)$$

where dE_i , dE_r and dE_w are the energy fluxes, respectively, incident on and reemitted from the surface, and the flux which would be re-emitted if all incident molecules were reemitted with a Maxwellian distribution corresponding to the surface temperature T_w . The quantity α then is a measure of the degree to which the incident molecules are "accommodated" to the surface. Measured values of α are usually slightly less than unity (Ref. 9). It will be observed that all quantities on the right side of Eq. 2.7 are well-defined and can be either calculated or measured.

The definition of σ , however, is somewhat less satisfactory. It is supposed that the incident molecules are either reflected "specularly" (i.e., with simple reversal of normal velocity) or "diffusely" (which for some applications can be just "randomly", but eventually must be defined more precisely as Maxwellian corresponding to a temperature T_r which is not necessarily the same as T_w). The coefficient σ is then defined as the fraction of diffusely reflected molecules, $1 - \sigma$ being the fraction which are specularly reflected. The

definition in this form of the quantity $\bar{\sigma}$ thus becomes meaningless unless the actual interaction with the surface is of the type specified. As measured values of $\bar{\sigma}$ have actually been determined, however, this difficulty has been avoided because only gross tangential forces are dealt with. Thus, according to the definition of $\bar{\sigma}$, one has,

$$\bar{\sigma} = \frac{\bar{\tau}_i - \bar{\tau}_r}{\bar{\tau}_i} \quad (2.8)$$

where $\bar{\tau}_i$ and $\bar{\tau}_r$ are the incident and reflected fluxes of tangential momentum (shear stress). For all practical purposes, Eq. 2.8, rather than the specular-diffuse model, has been the defining equation for $\bar{\sigma}$. The two definitions are in agreement if the interaction actually is of the specular-diffuse type (which it, of course, almost certainly is not), otherwise Eq. 2.8 defines $\bar{\sigma}$. Eq. 2.8 may be written also, for symmetry with Eq. 2.7, in the form:

$$\bar{\sigma} = \frac{\bar{\tau}_i - \bar{\tau}_r}{\bar{\tau}_i - \bar{\tau}_w} \quad (2.9)$$

where $\bar{\tau}_w$ is defined as the tangential momentum flux which would be reflected from the surface if all incident molecules were reemitted with a Maxwellian distribution corresponding to the surface temperature T_w . Clearly $\bar{\tau}_w = 0$.

Thus the difficulty inherent in the original definition of $\bar{\sigma}$ is avoided, insofar as the determination of tangential forces. However, it occurs again in the determination of normal forces, and again recourse has been taken in Refs. 1 and 2, for example, to the specular-diffuse model. If the actual interaction is different, then the determination of heat transfer, tangential force and normal force in terms of just the two parameters α and $\bar{\sigma}$ would yield results which one would expect to be mutually inconsistent. This difficulty can be avoided if an additional parameter $\bar{\sigma}'$, say, is defined by

$$\bar{\sigma}' = \frac{p_i - p_r}{p_i - p_w} \quad (2.10)$$

where p_i , p_r and p_w are respectively the incident and reflected fluxes of normal momentum (pressure) and the flux which would be reemitted if all the incident molecules were reemitted with a Max-

wellian distribution corresponding to the surface temperature T_W .

The quantity P_W is given by (Ref. 2)

$$P_W = N_W m \sqrt{\frac{\pi R T_W}{2}} \quad (2.11)$$

where N_W is the number of molecules per unit time incident on a unit surface area. It is given by

$$N_W = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} u' f du' dv' dw' \quad (2.12)$$

For completely specular reflection one has $\alpha = \sigma = \sigma' = 0$. For completely diffuse reflection, $\alpha = \sigma = \sigma' = 1$. For the hypothetical partly diffuse, partly specular interaction, and for a surface at rest with respect to the gas, one would have

$$\sigma' = \frac{\sigma \sqrt{T_i} - \sqrt{\sigma(\sigma - \alpha)T_i + \sigma\alpha T_W}}{\sqrt{T_i} - \sqrt{T_W}} \rightarrow \infty \text{ as } T_W \rightarrow T_i \quad (2.13)$$

However, in general, for actual physical interaction the three parameters would be independent. They are sufficient, however, to determine free molecule flow heat transfer and aerodynamic force characteristics.

2.3 The Combined Expressions

The total force on a unit of surface area is analysed into a normal component p and a tangential component τ given by

$$\left. \begin{aligned} p &= p_i + p_r \\ \tau &= \tau_i - \tau_r \end{aligned} \right\} \quad (2.14)$$

With the help of Eqs. 2.9 and 2.10 these reduce to

$$\begin{aligned} p &= (2 - \sigma') p_i + \sigma' P_W \\ \tau &= \sigma \tau_i \end{aligned} \quad (2.15)$$

In terms of f , using Eqs. 2.11, 2.12, 2.1 and 2.2, one has

$$\left. \begin{aligned} p &= m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f \left[(2-\sigma') u'^2 + \sigma' \sqrt{\frac{\pi RT_w}{2}} u' \right] du' dv' dw' \\ \tau &= \sigma m \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u' v' f du' dv' dw' \end{aligned} \right\} \quad (2.16)$$

The total force on a unit length of the cylinder is obtained in terms of the drag force F_D and the lift force F_L . These are given by

$$\left. \begin{aligned} F_D &= \int_0^{2\pi} (-p \cos \theta + \tau \sin \theta) r d\theta \\ F_L &= \int_0^{2\pi} (-p \sin \theta - \tau \cos \theta) r d\theta \end{aligned} \right\} \quad (2.17)$$

Combining Eqs. 2.17, 2.16, 2.5 and 2.3, one has finally,

$$\begin{aligned} F_D &= \frac{\rho}{(2\pi RT)^{3/2}} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[- \left[(2-\sigma') u'^2 + \sigma' \sqrt{\frac{\pi RT_w}{2}} u' \right] \cos \theta \right. \right. \\ &\quad \left. \left. + \sigma u' v' \sin \theta \right) e^{-\frac{(-u' \cos \theta + v' \sin \theta - U)^2 + (-u' \sin \theta - v' \cos \theta)^2 + w'^2}{2RT}} \right. \\ &\quad \left(1 + \frac{1}{2pRT} \left[p_{xx} (-u' \cos \theta + v' \sin \theta - U)^2 + p_{yy} (-u' \sin \theta - v' \cos \theta)^2 + \right. \right. \\ &\quad \left. \left. + p_{zz} w'^2 - 2\tau_{xy} (-u' \cos \theta + v' \sin \theta - U)(-u' \sin \theta - v' \cos \theta) \right. \right. \\ &\quad \left. \left. - 2\tau_{xz} (-u' \cos \theta + v' \sin \theta - U)w' - 2\tau_{yz} (-u' \sin \theta - v' \cos \theta)w' \right. \right. \\ &\quad \left. \left. - 2 \left(1 - \frac{(-u' \cos \theta + v' \sin \theta - U)^2 + (-u' \sin \theta - v' \cos \theta)^2 + w'^2}{5RT} \right) \left(q_x (-u' \cos \theta + v' \sin \theta - U) \right. \right. \right. \\ &\quad \left. \left. \left. + q_y (-u' \sin \theta - v' \cos \theta) + q_z w' \right) \right] \right\} r du' dv' dw' d\theta \end{aligned} \quad (2.18)$$

$$F_L = (2\pi RT)^{3/2} \int_0^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \left\{ - \left(\left[(2-\sigma') U^2 + \sigma' \sqrt{\frac{\pi RT}{2}} U \right] \sin \theta + \sigma U' v' \cos \theta \right) \times \right. \\ \left. e^{-\frac{(-U' \cos \theta + v' \sin \theta - U)^2 + (-U' \sin \theta - v' \cos \theta)^2 + w'^2}{2 RT}} \times \left(1 + \frac{1}{2 p RT} \left[p_{xx} (-U' \cos \theta + v' \sin \theta - U)^2 + p_{yy} (-U' \sin \theta - v' \cos \theta)^2 + p_{zz} w'^2 \right. \right. \right. \\ \left. \left. - 2 \tau_{xy} (-U' \cos \theta + v' \sin \theta - U)(-U' \sin \theta - v' \cos \theta) - 2 \tau_{xz} (-U' \cos \theta + v' \sin \theta - U) w' \right. \right. \\ \left. \left. - 2 \tau_{yz} (-U' \sin \theta - v' \cos \theta) w' - 2 \left(1 - \frac{(-U' \cos \theta + v' \sin \theta - U)^2 + (-U' \sin \theta - v' \cos \theta)^2 + w'^2}{5 RT} \right) \times \right. \right. \\ \left. \left. \left(q_x (-U' \cos \theta + v' \sin \theta - U) + q_y (-U' \sin \theta - v' \cos \theta) + q_z w' \right) \right) \right\} r du' dv' dw' d\theta \quad (2.19)$$

3.0 RESULTS

Performing the integration indicated in Eqs. 2.18 and 2.19, one obtains

$$F_D = \rho r U^2 \left\{ \frac{(4-2\sigma'+\sigma)\sqrt{\pi} e^{-\frac{S^2}{2}}}{6S} \left[(3+2S^2) I_0\left(\frac{S^2}{2}\right) + (1+2S^2) I_1\left(\frac{S^2}{2}\right) \right] \right. \\ \left. + \frac{\sigma' \pi^{\frac{3}{2}}}{4 S_w} + \frac{(4-2\sigma'+\sigma)\sqrt{\pi} e^{-\frac{S^2}{2}}}{60S} \left[\frac{6 q_x}{p U} \left(I_0\left(\frac{S^2}{2}\right) - I_1\left(\frac{S^2}{2}\right) \right) \right. \right. \\ \left. \left. + \frac{5 p_{xx}}{p} \left(2 I_0\left(\frac{S^2}{2}\right) + \left(2 + \frac{1}{S^2}\right) I_1\left(\frac{S^2}{2}\right) \right) \right. \right. \\ \left. \left. + \frac{5 p_{yy}}{p} \left(I_0\left(\frac{S^2}{2}\right) + \left(1 - \frac{1}{S^2}\right) I_1\left(\frac{S^2}{2}\right) \right) \right] \right\} \quad (3.1)$$

$$F_L = \rho r U^2 \frac{(4-2\sigma'+\sigma)\sqrt{\pi} e^{-\frac{S^2}{2}}}{60S} \left\{ \frac{10 \tau_{xy}}{p} \left[I_0\left(\frac{S^2}{2}\right) + \left(1 - \frac{1}{S^2}\right) I_1\left(\frac{S^2}{2}\right) \right] \right. \\ \left. + \frac{6 q_y}{p U} \left[I_0\left(\frac{S^2}{2}\right) + I_1\left(\frac{S^2}{2}\right) \right] \right\} \quad (3.2)$$

where I_0 and I_1 are the modified Bessel functions and S is the molecular

speed ratio

$$S = \frac{U}{\sqrt{2RT}} = \sqrt{\frac{\gamma}{2}} M \quad (3.3)$$

It will be observed that the drag force for the non-uniform case differs from that for the uniform case by terms proportional, respectively, to the heat flux in the X-direction and to the deviation of the normal stresses from the hydrostatic. The lift force, which is zero for the uniform case, contains terms proportional to the shear stress and the heat flux in the y direction.

Eqs. 3.1 and 3.2 may be rewritten in the form

$$C_D \equiv \frac{F_D}{\rho U^2 r} = C_{D0} + \frac{q_x}{pU} C_{Dq_x} + \frac{p_{xx}}{p} C_{Dp_{xx}} + \frac{p_{yy}}{p} C_{Dp_{yy}} \quad (3.4)$$

$$C_L \equiv \frac{F_L}{\rho U^2 r} = \frac{\tau_{xy}}{p} C_{L\tau_{xy}} + \frac{q_y}{pU} C_{Lq_y} \quad (3.5)$$

The indicated partial lift and drag coefficients, $C_{L\tau_{xy}}$, C_{Dq_x} , etc., correspond to the contribution to the total lift or drag arising from the non-uniformity denoted in the subscript. These results are presented graphically in Figs. 2 and 3 for the case of diffuse reflection. It should also be mentioned that the results of Eqs. 3.1 and 3.2 agree with those of Ref. 2 for the uniform case.

Inspection of Figs. 2 and 3 reveals that the partial lift and drag coefficients corresponding to the non-uniform terms are all small compared to the coefficient for the uniform case C_{D0} . Unless a gas flow is very non-uniform, i.e., if the flow itself is in the continuum or slip flow region, the quantities p_{xx} , p_{yy} and τ_{xy} are small compared to p . It follows that the contribution to the total force on the cylinder arising from these viscous stress terms will be small compared to the force due to the uniform flow (of the order of a few per cent or less). These quantities might thus be neglected in connection with use of the cylinder as a probe.

The relative importance of the forces arising from the heat flux terms, however, depends on the magnitude of the ratio of q_x or q_y to pU . At high flow velocities these forces will thus be negligible compared to the force for the uniform case. At very low velocities they will not. In a stagnation region such forces constitute the entire force on the cylinder.

4.0 CONCLUSIONS

- 4.1 In the free molecule flow of a non-uniform gas (i.e., one in which heat flux or viscous stresses are present) past a cylinder, both the lift and drag forces will be affected by the non-uniformity.
- 4.2 Force contributions due to stress and heat flux terms are, in general, small compared to the force for the uniform case, except in regions of low flow velocity where the forces arising from the heat flux terms become important.

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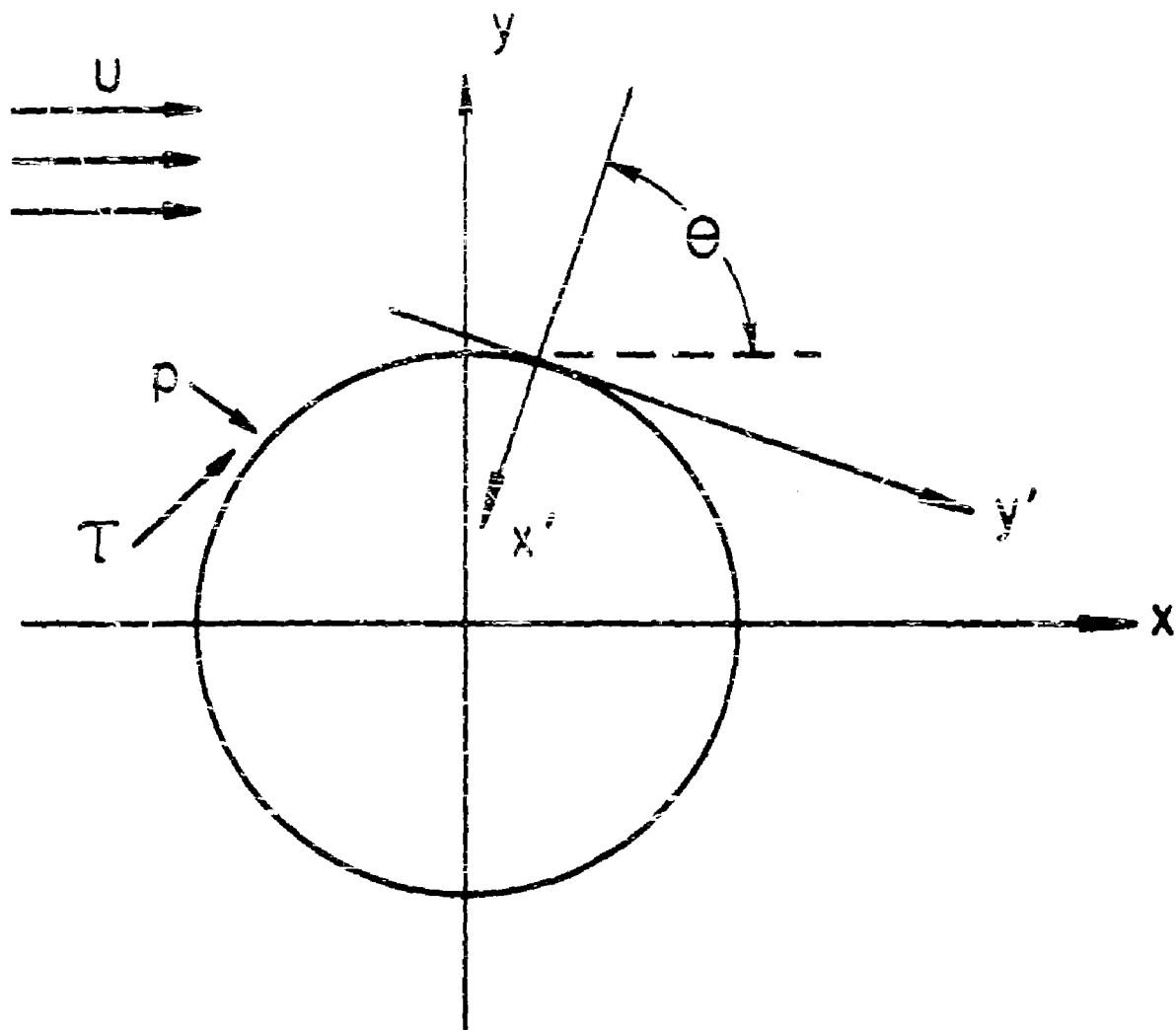


FIG.1 COORDINATE SYSTEM

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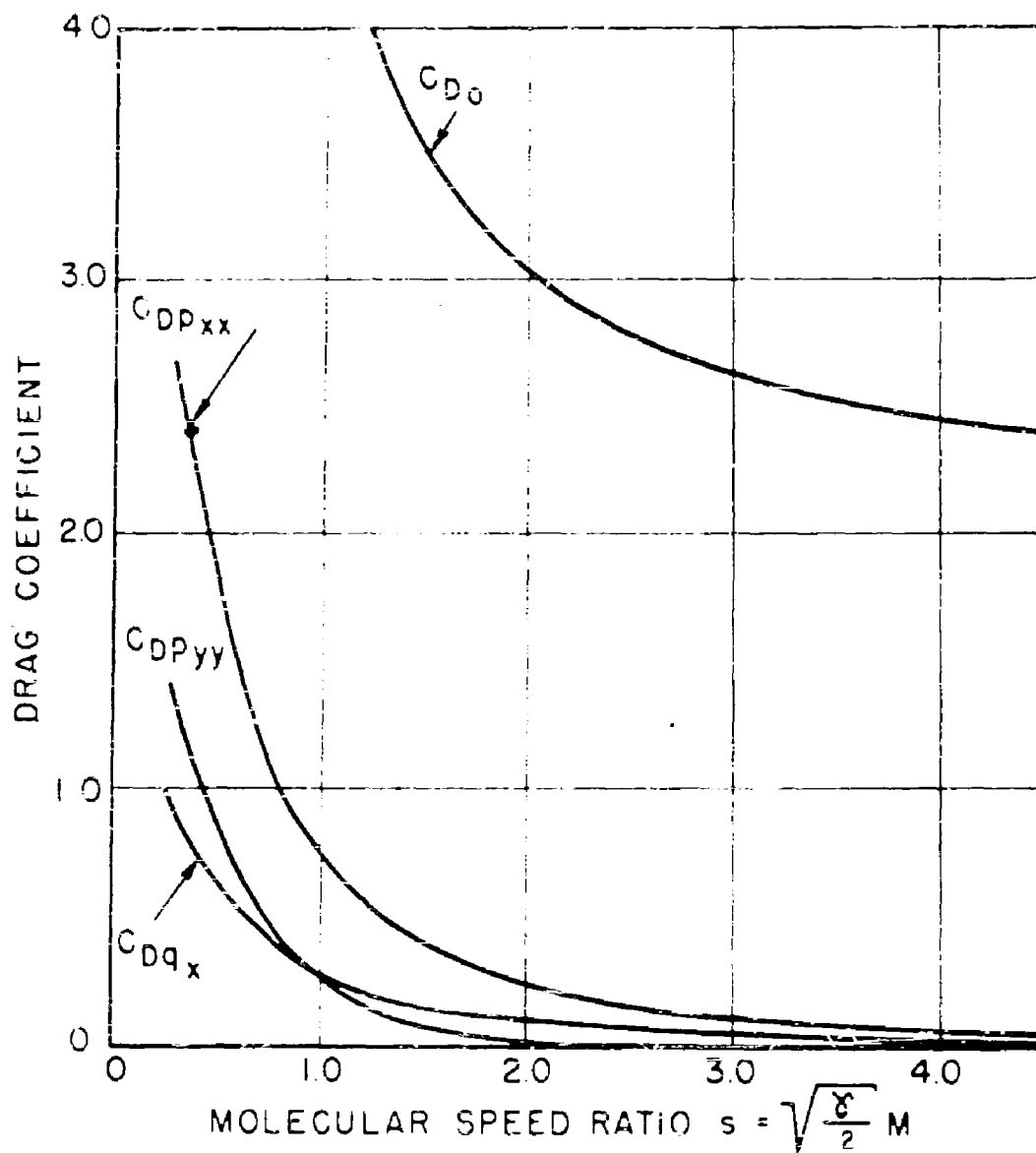


FIG. 2 PARTIAL DRAG COEFFICIENT FOR NON-UNIFORM FREE MOLECULE FLOW PAST A CYLINDER (DIFFUSE REFLECTION)

$$C_D = C_{D0} + \frac{p_{xx}}{p} C_{Dp_{xx}} + \frac{p_{yy}}{p} C_{Dp_y} + \frac{q_x}{pU} C_{Dq_x}$$

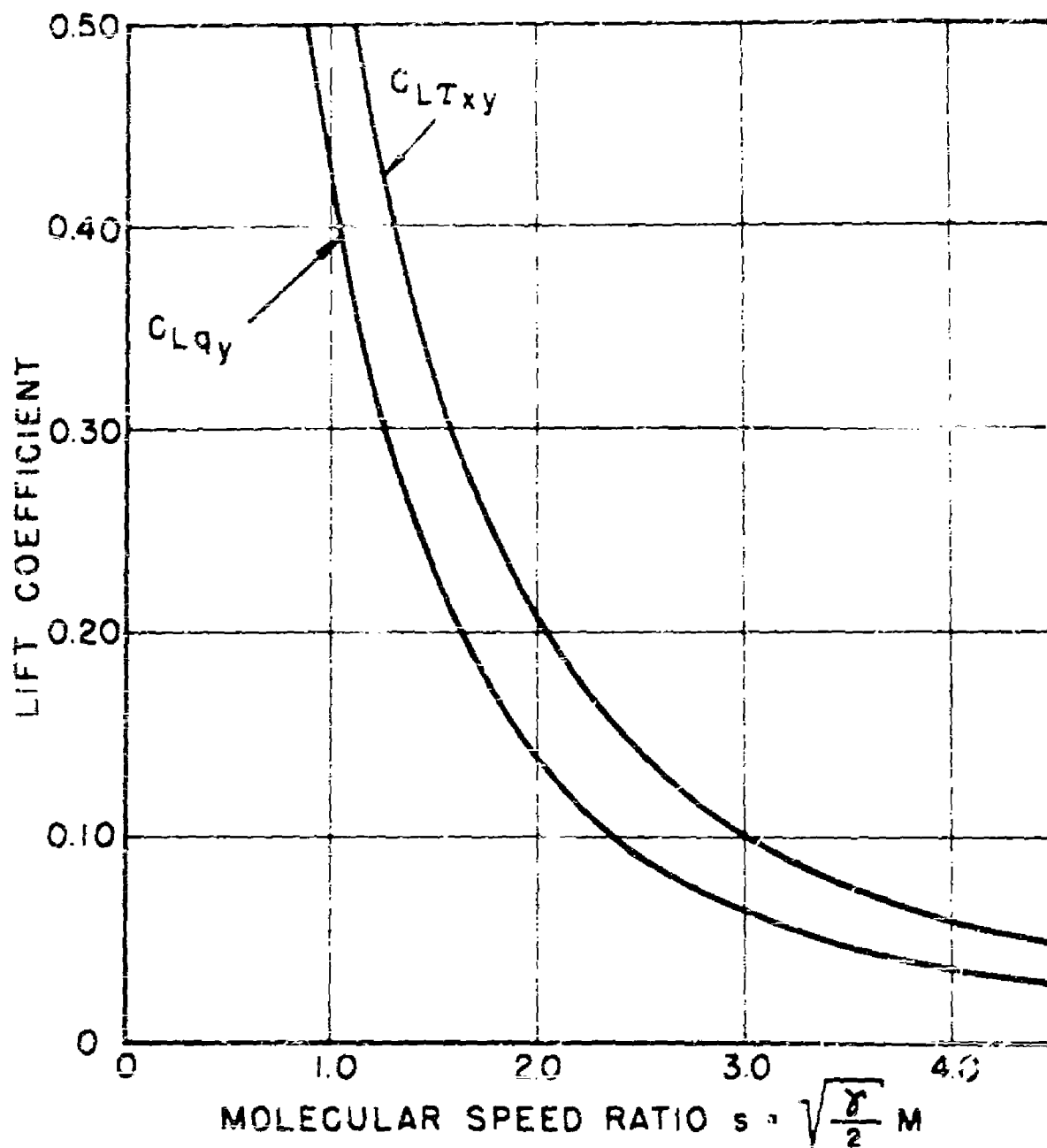


FIG. 3 PARTIAL LIFT COEFFICIENT FOR NON-UNIFORM FREE MOLECULE FLOW PAST A CYLINDER (DIFFUSE REFLECTION)

$$C = \frac{T_{xy}}{p} C_{LT_{xy}} + \frac{q_y}{p \cdot U} C_{Lq_y}$$

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